Life-Cycle Analysis with an Increasing Wage Profile

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(Preliminary and Incomplete)

Abstract

This paper investigates the effects of an increasing efficiency profile of labor documented by Rupert and Zanella (2010) on hours worked throughout the life cycle using a general equilibrium framework. Results show that the income effect dominates the substitution effect coming from higher wages, hence after making a peak at prime ages, hours worked start declining far before reaching retirement. These results are robust to different intertemporal elasticity of substitution for labor and different utility functions.

1 Introduction

This study aims to investigate the effects of an increasing labor efficiency over the life cycle on hours worked and explain why households decrease their hours of work sharply towards retirement. Life-cycle profiles of wages, hours and earnings have been extensively discussed in the macro-labor literature, and a hump-shaped earnings profile is common in the literature. This is mostly attributed to life-cycle wage profile being hump-shaped, especially in the human capital models, where the wage rate is defined as the return on one's

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human capital (Ben-Porath (1967); Heckman (1976)). French (2005) and Manovskii and Kambourov (2005) use Panel Study of Income Dynamics (PSID) data in their empirical work and come up with a hump-shaped wage profile as in the human capital literature. Hansen (1993) constructs an efficiency units profile using BLS data, and finds that returns to unit labor form a hump-shape over the life-cycle too.

While the studies mentioned above documents the evolution of wages over the life cycle, other studies look at how hours worked respond to changes in wages. Pencavel (1986) asserts that individuals supply more hours when their wage rate is the highest in their life cycle. In another paper, Pencavel (2002) estimates the intertemporal substitution elasticity with respect to wage which measures the response of work hours to evolutionary changes in wages throughout the life cycle, and finds that the elasticity changes between 0.15 and 0.28 whereas Macurdy (1981) estimates it as being between 0.1 and 0.23. Devereux (2003), on the other hand, estimates the same elasticities, and conclude that wage effects on labor supply are too small to explain the variations in labor supply.

In their working paper, Rupert and Zanella (2010) uses PSID data to show that the wage profile is actually not decreasing over the life-cycle. They generate a hump-shaped earnings profile, which is driven by a sharp decrease in the hours worked after mid 50's. This study aims to investigate the life-cycle profiles of hours worked in a general equilibrium framework taking the increasing wage profile from PSID data as exogenously given, and explain the sharp decrease in hours worked towards retirement.

2 Empirical Life-Cycle Profiles

To get the life-cycle profiles for wages, earnings and hours worked, I closely follow Rupert and Zanella (2010). PSID data identifies each household and individual with a unique interview ID each year. This feature of the data enables us to follow a set individuals throughout almost all their working lives. The cohort I follow is the 23 year-old cohort, as in Rupert and Zanella (2010). Only the male head of households who work for someone else are observed for this purpose. This gives us a more homogenous group. The profiles are generated using averages of individuals at each age using the PSID sample weights. Using these weights makes the data more representative of the U.S. population. Here I follow two different strategies to follow the individuals over their life-cycle. First, I take all individuals who are 23 years old and entered the market in 1967. PSID data starts in 1968, but since the individuals report their incomes, hours worked and wages from the previous year, 1967 is the first year we observe. Following Rupert and Zanella (2010) again, I take 5-year bins for each age such that age 23 reports values for individuals who were aged between 21 and 25. So for each year, I use the middle value of the 5 year bin to represent the age of the individual. This method gives us more observations as the sample size is small otherwise, and decreases over time as age increases. Then I follow these individuals throughout their life cycles. As I mentioned above, the sample size drops over time, due to death or other reasons. And among those, I take the averages of individuals who supplied positive work hours. Figure 1 illustrates the life-cycle profiles of the 23 year-old cohort. All nominal values are converted to real by using CPI-U, taking $1982 - 1984 = 100$. The bottom-right graph in Figure 1 depicts the percentage of respondents who answered the question “Do you have any physical or nervous condition that limits the type of work or the amount of work that you can do?” with yes in the PSID family surveys.

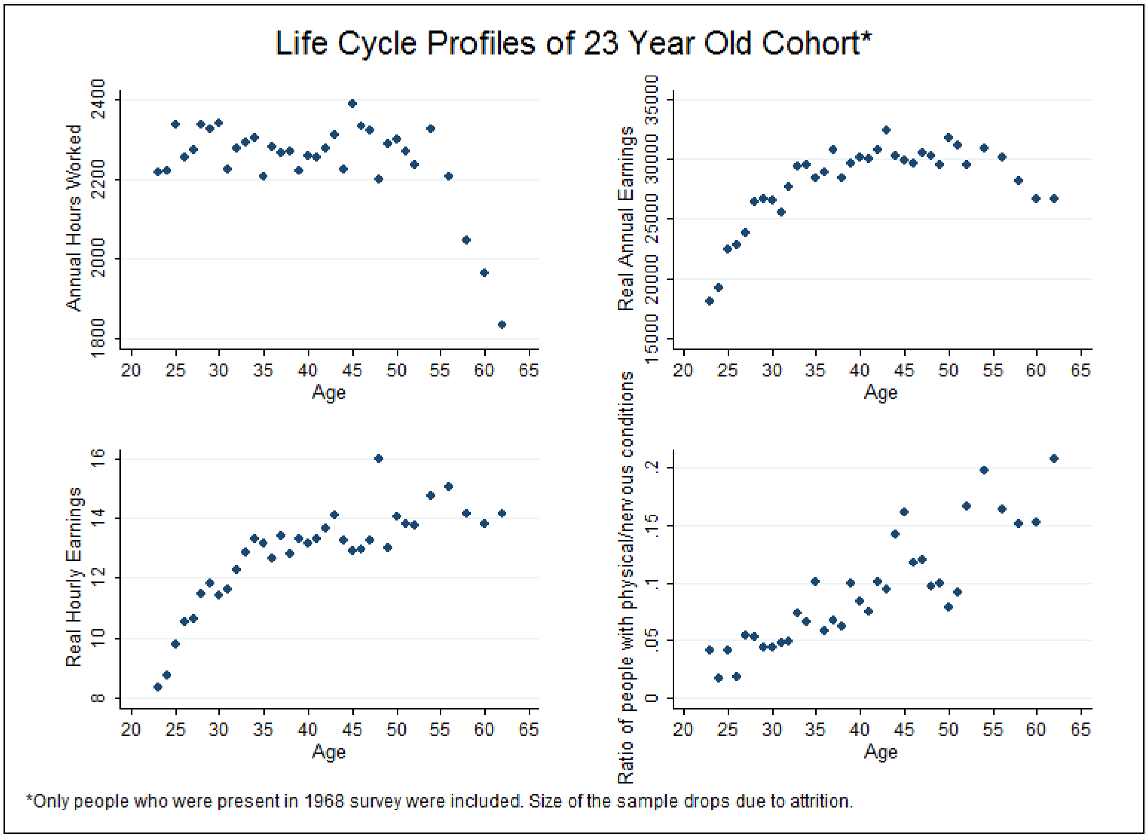


Figure 1: Using Method 1

As a second method, I follow those households who are present in all the survey years between 1968 and 2007. Using the same selection criteria as in the first method, and applying the 5-year bins, there are 106 such individuals who are present in all years. The profiles for this method are shown in Figure 2.

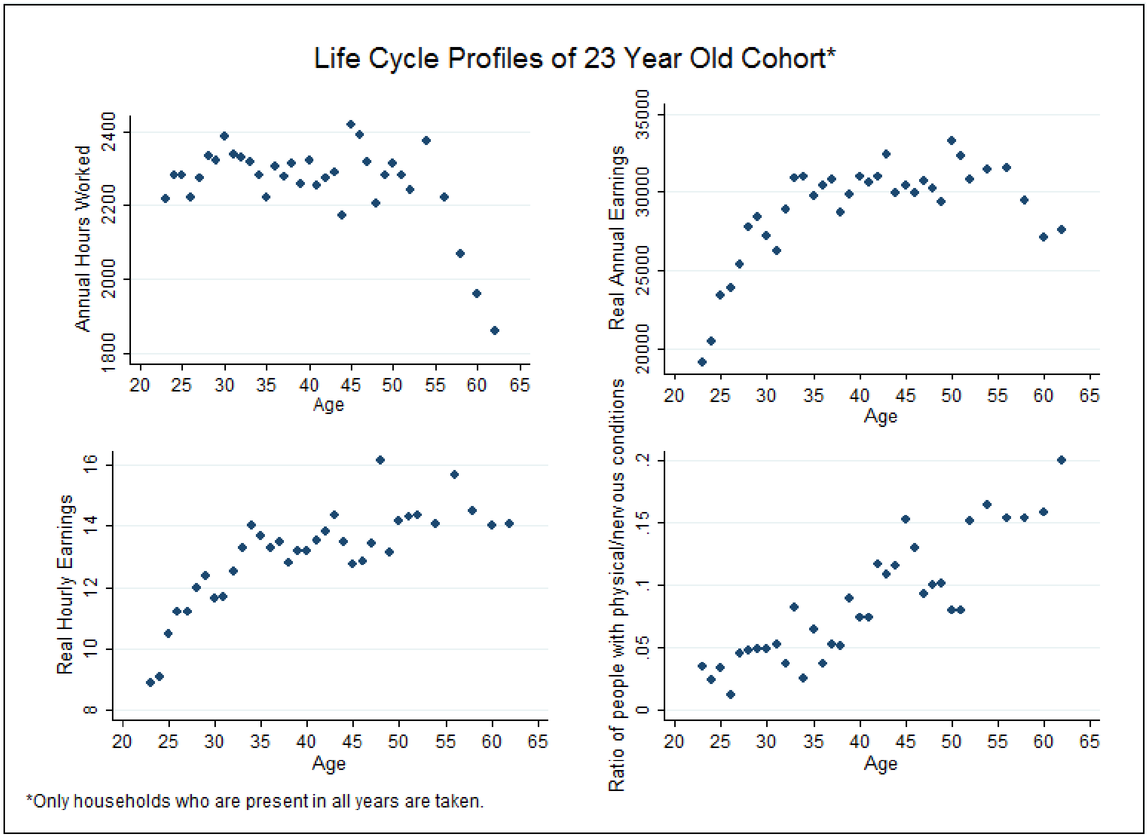


Figure 2: Using Method 2

As we can clearly see, methods 1 and 2 give almost the same life cycle profiles. I am going to continue with the profiles obtained from method 2 from this point on.
As seen in Figure 2, real annual earnings exhibit a hump-shape, rapidly increasing during the first 10 years of the working life, staying constant until 50's, and declining thereafter. We cannot observe such a decline in the real hourly earnings over the life cycle though. The profile for real hourly earnings is increasing with age, with diminishing returns. When we look at the profile for annual hours worked, we see a more or less stable patter until 50's, but then hours worked decline sharply. And lastly the ratio of people whose work hours are limited due to a physical or nervous condition steadily increases with age as seen in

the last panel. One might ask whether this self-reported measure of health status reflects the true health status of an individual or not. Benitez-Silva et al. (2004) show that self-reported health status is a good indicator of the true health status of an individual, and it affects the labor force participation of the individual. Cai (2010) also finds that health has a positive and significant effect on labor supply, meaning that healthier people supply more labor. Stern (1989) shows that disability of the worker adversely affects his/her labor force participation too.

3 Model

3.1 Demographics

The economy is populated by overlapping generations of individuals of age $j = 1, 2, ..., J$. J_R is the mandatory retirement age. One model period is equivalent to one year. Population growth rate is assumed to be zero. Conditional probability of surviving from age $j-1$ to age j is denoted by ρ_j, with $\rho_1 = 1$ and $\rho_J = 0$. Cohort shares are constant and equal to μ_j. Given the sequence of conditional survival probabilities, $\{\rho_j\}_{j=1}^{J}$, time invariant cohort shares, $\{\mu_j\}_{j=1}^{J}$ can be determined by:

$$\mu_{j+1} = \rho_{j+1}\mu_j \tag{1}$$

And since $\sum_{j=1}^{J} \mu_j = 1$, μ_1 can be determined by using this and (1). Since we are interested in steady states, all time subscripts are omitted below.

3.2 Preferences

Households maximize the following lifetime utility:

$$\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{k=1}^{j} \rho_k \right) u(c_j, h_j) \tag{2}$$

where β is the subjective time discount factor, c_j and h_j are consumption and hours worked by the age j household.

The utility function is assumed to be separable for the benchmark model, and is of the following form:

$$u(c_j, h_j) = \frac{c_j^{1-\sigma}}{1-\sigma} - \psi \frac{h_j^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \tag{3}$$

where σ is the coefficient of relative risk aversion, γ captures the intertemporal elasticity of substitution in labor, and ψ is the weight on disutiliy from work.

Households are endowed with one unit of time each period, which can be allocated between work and leisure until retirement. h_j is zero after the mandatory retirement age. Households' earnings are given by $w\epsilon_j h_j$ for $j = 1, 2, ...J_R - 1$ where w is the market wage rate, and ϵ_j is the age specific efficiency unit of labor. For $j = J_R, ..., J$, households simply earn a fixed benefit, b. Accidental bequests due to death are redistributed to all living households in a lump-sump manner at the amount T.

3.3 Technology

There is a representative firm that has access to a constant returns to scale, Cobb-Douglas production function of the form $Y = BK^{\alpha}N^{1-\alpha}$, with $B > 0$, and the capital's share of output, $\alpha \in (0, 1)$. Capital depreciates at a constant rate, $\delta \in (0, 1)$. Firm rents capital and hires labor from households, paying them the marginal products of capital and labor:

$$r = \alpha B\Big(\frac{K}{N}\Big)^{\alpha-1} - \delta \tag{4}$$

$$w = (1-\alpha)B\Big(\frac{K}{N}\Big)^{\alpha} \tag{5}$$

3.4 Social Security

There is a pay-as-you-go social security system, where households are taxed by the rate τ_{ss} on their labor income, and the benefits that the retired households get are fully funded by these tax revenues. Following Imrohoroglu et al. (1995), the benefits that the retired households get are are defined as a proportion of their average lifetime earnings from working, which is given as:

$$b = \theta \frac{\sum_{j=1}^{J_R-1} w\epsilon_j h_j}{J_R - 1} \tag{6}$$

where θ is the replacement ratio.

3.5 Market structure

Households cannot insure against the mortality risk, thus the markets are incomplete. However, they are allowed to hold one-period riskless bonds, a_{j+1}. There are borrowing constraints in the economy, so $a_{j+1} \geq 0$ for all j.

3.6 Households' Problem

Households are heterogeneous in their ages and asset holdings. The optimal allocations of the household is computed by solving the following dynamic problem recursively.

$$V(a_j) = \max_{c_j, h_j, a_{j+1}} \left\{ u(c_j, h_j) + \beta \rho_j V(a_{j+1}) \right\} \tag{7}$$

subject to

$$c_j + a_{j+1} = (1 + r)a_j + (1 - \tau_{ss}) w \epsilon_j h_j + T \qquad \text{for} j = 1, 2, ..., J_R - 1 \tag{8}$$

$$c_j + a_{j+1} = (1 + r)a_j + b + T \qquad \text{for} j = J_R, ..., J \tag{9}$$

$$a_{j+1} \geq 0 \tag{10}$$

3.7 Competitive Equilibrium

For given demographic parameters $\{\rho_j\}_{j=1}^{J}$, a stationary competitive equilibrium consists of households' decision rules $\{c_j, h_j, a_{j+1}\}_{j=1}^{J}$, factor prices w and r, social security tax rate τ_{ss}, accidental bequests T, and constant cohort shares $\{\mu_j\}_{j=1}^{J}$ that satisfy that following conditions:

1. Households' decision rules solve (7) subject to (8), (9) and (10).
2. Factor prices are determined competitively:

$$r = \alpha B \left(\frac{K}{N} \right)^{\alpha - 1} - \delta$$
$$w = (1 - \alpha) B \left(\frac{K}{N} \right)^{\alpha}$$

3. Lump-sum transfers of accidental bequests is equal to the amount of assets left by

the dead:

$$T = \sum_{j=1}^{J} \mu_j (1 - \rho_j) a_j$$

4. The labor and capital markets clear:

$$K_t = \sum_{j=1}^{J} \mu_j a_j$$

$$N_t = \sum_{j=1}^{J_R - 1} \mu_j \epsilon_j h_j$$

4 Calibration

First, we obtain the efficiency profile of labor using the PSID real hourly earnings profile, and compare it with the hump-shaped profile of efficiency of labor from Hansen (1993) and Hansen and Imrohoroglu (2009). Efficiency profile using PSID data is smoothed by using nonlinear least squares and predicting the fitted values. Figure 3 shows the fit, and Figure 4 shows the efficiency profiles of Hansen (1993) and PSID.

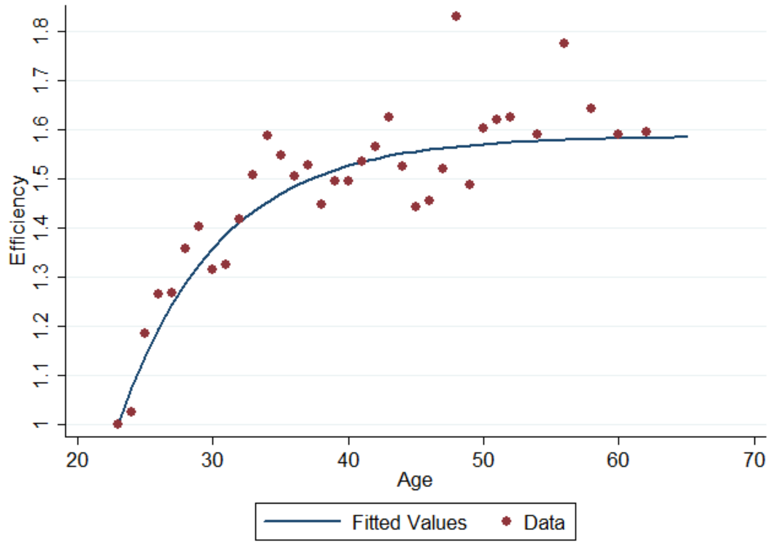
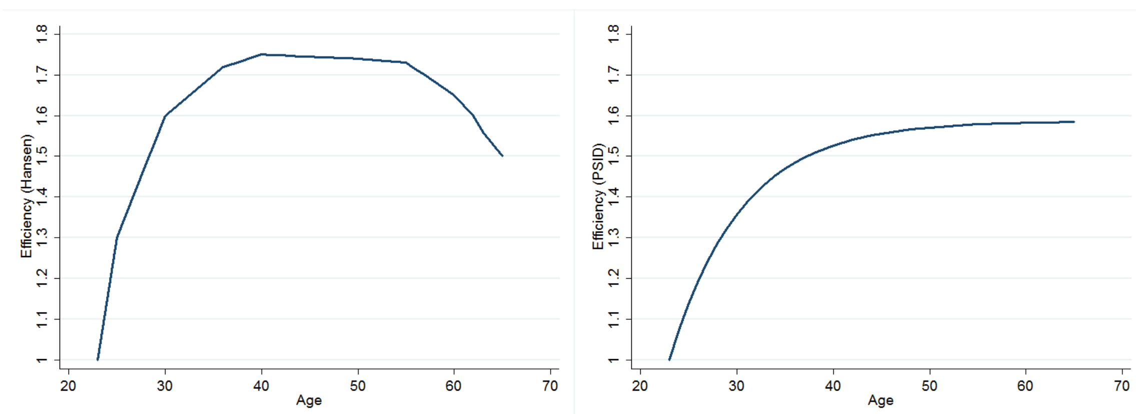


Figure 3: Fitted Values using NLS



(a) Hansen's Efficiency Profile

(b) Efficiency profile from PSID

Figure 4: Efficiency Profiles

Conditional survival probabilities for men are taken from National Vital Statistics Reports 2010. Profiles for conditional probability of surviving from age $j-1$ to age j and cumulative probability of surviving to age j are given in Figure 5

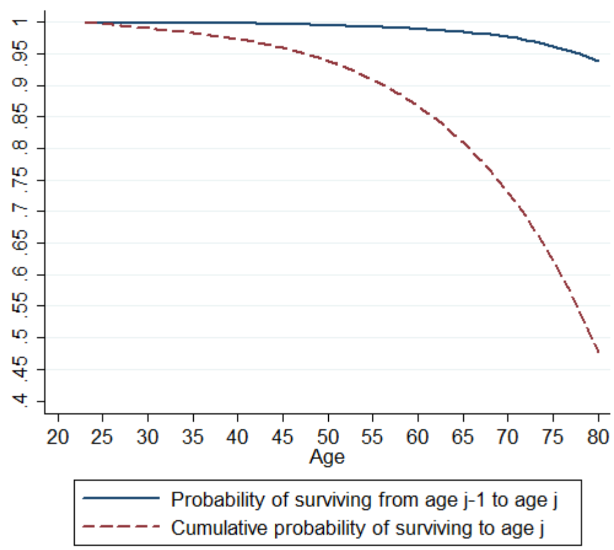


Figure 5: Survival Probabilities

Throughout the calibrations, an average hours worked ratio of around 0.38 was targeted to match the profile obtained from PSID data. And a capital-output ratio of around 3 was targeted to match the U.S. capital-output ratio. Mandatory retirement age was set as 65, and maximum lifetime possible is set to 80. Since we start the model at age 23, 65 corresponds to $J_R = 44$, and 80 corresponds to $J = 58$. The calibration process takes place as follows: I first choose values for parameters for the coefficient of relative risk aversion, intertemporal elasticity of substitution in labor, capital's share of output, depreciation rate of capital and social security replacement ratio from the literature. Then I calibrate the subjective time discount factor β and the weight on disutility from work ψ to match the targets. Initial guesses for wage, interest rate, social security tax rate, social security benefits and accidental bequests are given, and the quantitative model is iterated until these variables converge. I used two different values for the intertemporal elasticity of substitution in labor. Table 1 shows the choice of parameters, and calibrated parameters.

Preference Parameters	Explanation	Value	
α	Capital's share of output	0.36	
δ	Depreciation rate of capital	0.06	
θ	Social security replacement ratio	0.44	
σ	Coefficient of relative risk aversion	1 (log utility)	
γ	IES	0.1,0.5	
Calibrated Parameters		IES=0.1	IES=0.5
β	Subjective time discount factor	0.965	0.965
ψ	Weight on disutility from labor	35000	17

Table 1: Parameters

Figure 6 shows the results. We can see that the hours profile generated by the model with $\gamma = 0.1$ is relatively flat, with little change of hours over the life cycle, whereas the profile generated by the model with $\gamma = 0.5$ is above the profile with $\gamma = 0.1$ until mid 40's, and below thereafter. And also we can see that the variation in hours with $\gamma = 0.5$ is considerably more than the other one. Notice that both simulated hours profiles are unable to match the data in the sense that hours do not exhibit a sharp decrease after 50's. With $\gamma = 0.5$, even though the efficiency profile is an increasing one, income effect dominates the substitution effect due to higher labor earnings, and hours begin to decline after the first 10 years of the life cycle.

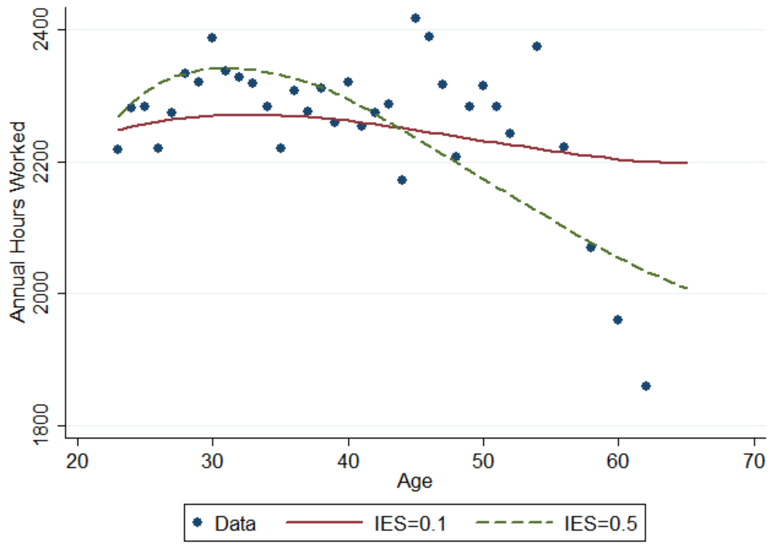
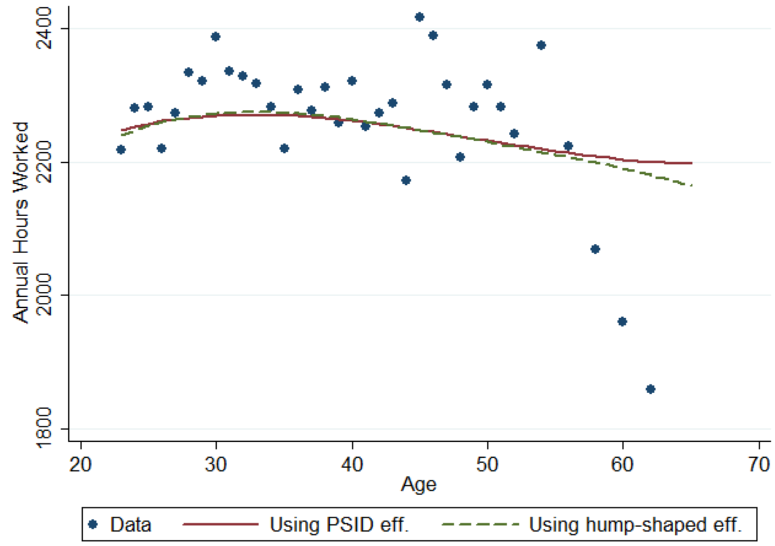
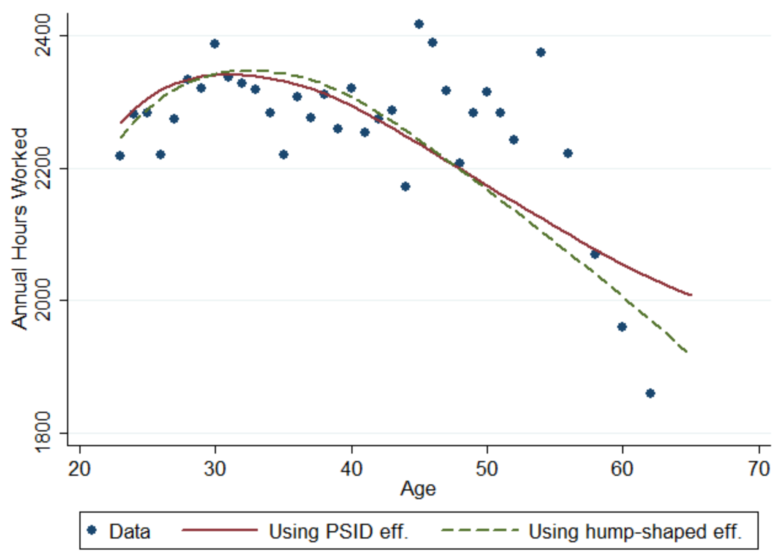


Figure 6: Simulated Hours vs. Data

Next we use the hump-shaped efficiency profile from Hansen (1993) to see how that effects the profiles for hours worked over the life cycle. Parameters remain same as in Table 1. And results are shown in Figure 7. We can see that the hours profile generated using the hump-shaped efficiency profile is more humped relative to the one generated using the increasing efficiency profile, which means compared to a hours profile with increasing efficiency of labor, households work less in the first few years of their life cycles, then work more until about mid 40's, and then work less again for the remaining part of their life cycles when they have a hump-shaped labor efficiency profile. The picture is qualitatively the same for different values of γ, but we can see from the top and the bottom panel of Fifgure 7 that the hours profiles become more humped when γ increases.



(a) $\gamma = 0.1$



(b) $\gamma = 0.5$

Figure 7: Increasing vs. Hump-shaped Efficiency

4.1 Different Utility Functions

So far we have assumed that the utility function is separable in consumption and leisure. This section investigates the effects of using a non-separable utility function on the life cycle profile of hours worked. We consider the following non-separable utility function.

$$u(c_j, h_j) = \frac{[c_j^{\nu}(1-h_j)^{1-\nu}]^{1-\sigma}}{1-\sigma} \tag{11}$$

where σ still represents the coefficient of relative risk aversion, and ν represents the weight of consumption in utility. Notice that for $\sigma = 1$ we have a log utility. Table 2 shows the calibrated parameters and Figure 8 displays the results of the model generated hours.

Preference Parameters	Explanation	Value
α	Capital's share of output	0.36
δ	Depreciation rate of capital	0.06
θ	Social security replacement ratio	0.44
σ	Coefficient of relative risk aversion	1 (log utility)
Calibrated Parameters		
β	Subjective time discount factor	0.9705
ν	Utility weight on consumption	0.41

Table 2: Parameters for non-separable utility

We can see that using a non-separable utility function generated a more humped profile compared to those with a separable utility function. Qualitatively the results remain the same though: An increasing efficiency profile induces households to work more in the first few years and after late 40's-early 50's compared to a hump-shaped efficiency profile.

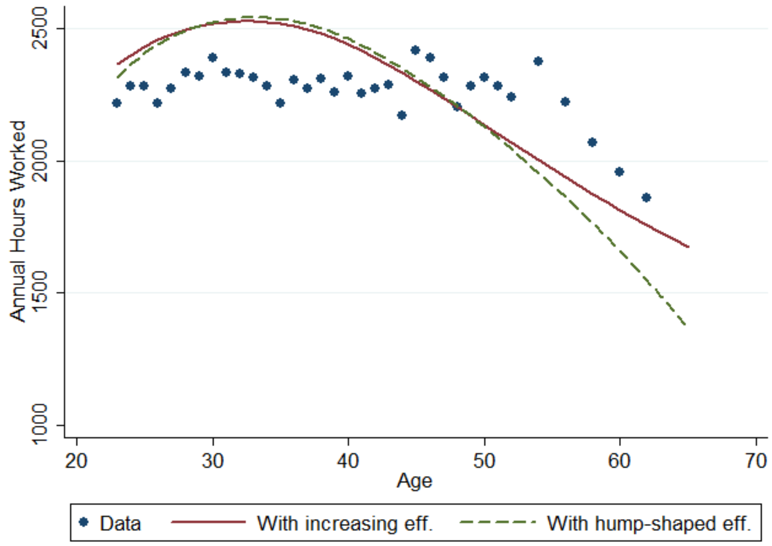


Figure 8: Simulated Hours vs. Data (Non-Separable Utility)

5 Conclusion

In this study, I examine how life cycle profile for hours worked behaved when the efficiency units of labor increased steadily over the life cycle. Whether I use a separable or non-separable utility function, the life cycle hours worked profiles I get have a hump-shape, where households start decreasing their work hours in the earlier periods of their life cycles, due to income effect. On the other hand, the profiles generated by the model in this study show that in all cases, using an increasing efficiency profile makes the hours worked profiles less humped. Lower values of intertemporal elasticity of substitution for labor generate less humps in the hours worked profile.

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